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Generalized quantum telecloning

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Abstract. We present a generalized telecloning (GTC) protocol where the quantum channel is non-optimally entangled and we study how the fidelity of the telecloned states depends on the entanglement of the channel. We show that one can increase the fidelity of the telecloned states, achieving the optimal value in some situations, by properly choosing the measurement basis at Alice's, albeit turning the protocol to a probabilistic one. We also show how one can convert the GTC protocol to the teleportation protocol via proper unitary operations.

PACS. 03.67.Mn Entanglement production, characterization, and manipulation – 03.67.Hk Quantum communication – 03.67.-a Quantum information

1 Introduction

Since the appearance of the quantum teleportation protocol [1] and its experimental demonstration [2,3], whereby an arbitrary state describing a quantum system can be transferred from one recipient (Alice) to another (Bob), several new quantum communication protocols have appeared. They allow the sharing of quantum states among several recipients [4], the sharing of quantum secrets [5–7], or the teleportation of an arbitrary quantum state to many recipients, i.e. quantum telecloning [8]. The latter protocol does not violate the no-cloning theorem [9] since the fidelity of the telecloned states with respect to the original one are not perfect, and decreases with the number of copies. An optimal quantum telecloning protocol has been presented in references [8,10] for two-level systems (qubits) and later on quantum telecloning has been demonstrated experimentally for continuous variables systems [11,12].

These protocols are essential to many quantum information tasks which require a secure transmission of quantum states. One example is quantum information networks [4,13], which are built of nodes in which quantum states are created, manipulated, and stored. These nodes are connected by multipartite entangled quantum channels and by properly using one or several of the aforementioned protocols one could avoid errors and eavesdropping during the transmission of a state between nodes [4,14].

However, most treatments of these protocols assume bipartite or multipartite maximally entangled channels, whereas in realistic scenarios decoherence and noise ensure that that is not the case. One suggested solution is quantum distillation protocols [15], which allow us to obtain a maximally entangled state from a large ensemble of partially entangled states, although only asymptotically. Another one is to dynamically control the decoherence of the channel qubits [16,17].

In reference [18], inspired by reference [19], and in reference [20], we have generalized the teleportation [18] and quantum state sharing [20] protocols to an arbitrary number of input qubits and shown that one can overcome the fidelity decrease due to non-maximally entangled channels on expense of transforming the protocols to probabilistic ones. These generalized protocols give the parties freedom to allocate the channel's resources to a continuous distribution between the fidelity of the protocol and its probability of success to achieve a given fidelity. Other interesting approaches using pure non-maximally entangled resources were presented in references [21–23]. In reference [21] it was shown how to directly teleport a qubit using non-maximally entangled pure channels. Contrary to reference [19], in reference [21] Bob needs to implement a unitary operation on his qubit and an ancillary plus a measurement on the ancillary in order to finish the protocol. In reference [22] it was discussed how to implement entanglement swapping using non-maximally pure entangled states and in reference [23] how to construct an oblivious remote state preparation procedure using non-maximally entangled resources.

In this contribution we present the generalized telecloning protocol (GTC), where we generalize the standard quantum telecloning protocol to non-optimally entangled multipartite channels (see Fig. 1). For a comprehensive review of other interesting extensions of the telecloning protocol see reference [24]. By treating each qubit's degraded

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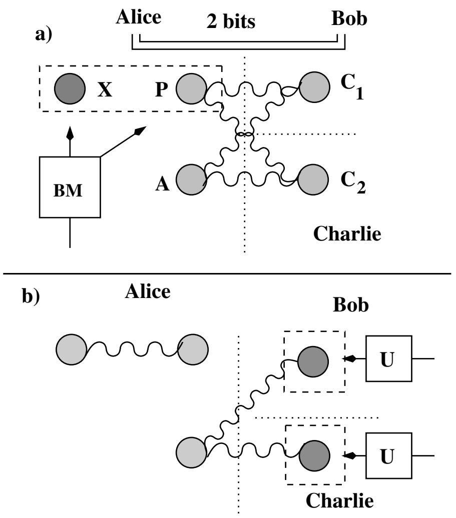

Fig. 1. Alice performs a Bell measurement (BM) on the qubit to be telecloned (X) and on the port qubit (P). She then tells Bob and Charlie her measurement result (2 bits). The copies C_1 and C_2 are then subjected to a proper unitary operation (U). Note that waves represent the existence of pairwise entanglement among the qubits and that the ancillary qubit (A) is entangled with the copies at the end of the protocol.

contribution to the entanglement of the channel separately, we show that one can overcome the resulting fidelity decrease by applying appropriate modifications to the protocol. Our main results show that: (a) the port's qubit influence on the entanglement of the channel can be overcome by changing the measurement basis; (b) the ancillary qubit's behavior has no effect on the telecloned fidelity; (c) the copy qubits' behavior has a non trivial influence on the fidelity of the telecloned states and we show the optimal strategy to maximize the efficiency of the protocol; and (d) it is possible to convert the GTC to the generalized teleportation protocol (GTP) if one allows Alice to implement certain types of unitary operations on the channel's qubits.

2 General formalism

We focus our attention on the "1 → 2 quantum telecloning", i.e. one original qubit and two copies. Let us assume that Alice wishes to teleclone her state to Bob and Charlie. The quantum channel used for the optimal telecloning protocol [8,10] is composed of four qubits, namely port qubit, ancillary qubit and two copy qubits. The port and ancillary qubits are assumed to be with Alice, although the ancillary is not required to be there [8]. One copy qubit is with Bob while the other one is with Charlie (Fig. 1).

The channel state is given by:

$$|\psi\rangle_{PAC} = \frac{1}{\sqrt{2}}\left(|0\rangle_P \otimes |\phi_0\rangle_{AC} + |1\rangle_P \otimes |\phi_1\rangle_{AC}\right), \quad (1)$$

where

$$|\phi_0\rangle_{AC} = \sum_{j=0}^{1} \alpha_j |\{0,1-j\},\{1,j\}\rangle_A \otimes |\{0,2-j\},\{1,j\}\rangle_C, \quad (2)$$

$$|\phi_1\rangle_{AC} = \sum_{j=0}^{1} \alpha_j |\{0,j\},\{1,1-j\}\rangle_A \otimes |\{0,j\},\{1,2-j\}\rangle_C, \quad (3)$$

$$\alpha_j = \sqrt{(2-j)/3}. \quad (4)$$

Here the subscripts denote the port (P), ancillary (A) and copies (C: C_1 with Bob and C_2 with Charlie). The state $|\{0,M-j\},\{1,j\}\rangle$ represents the symmetric and normalized state of M qubits in which $M-j$ of them are in the state $|0\rangle$ and j are in the orthogonal state $|1\rangle$ (see Ref. [8]). For $M=2$ we have explicitly,

$$|\phi_0\rangle_{AC} = \sqrt{\frac{2}{3}}|000\rangle_{AC} + \sqrt{\frac{1}{6}}|101\rangle_{AC} + \sqrt{\frac{1}{6}}|110\rangle_{AC},$$

$$|\phi_1\rangle_{AC} = \sqrt{\frac{2}{3}}|111\rangle_{AC} + \sqrt{\frac{1}{6}}|001\rangle_{AC} + \sqrt{\frac{1}{6}}|010\rangle_{AC}.$$

We analyze the influence of each qubit on the entanglement of the channel by applying a qubit-specific 'disentanglement' operator:

$$\hat{D}_i\left(\alpha|0\rangle_i|\psi_0\rangle + \beta|1\rangle_i|\psi_1\rangle\right) = \frac{\alpha|0\rangle_i|\psi_0\rangle + n_i\beta|1\rangle_i|\psi_1\rangle}{\sqrt{|\alpha|^2 + |n_i\beta|^2}}, \quad (5)$$

where n_i can be complex and $|\alpha|^2 + |\beta|^2 = 1$. For example, when this operator is applied on a Bell state, e.g. $|\Phi^+\rangle = 1/\sqrt{2}\,(|00\rangle + |11\rangle)$, it produces a non-maximally entangled state, $\hat{D}_1(|\Phi^+\rangle) = 1/\sqrt{1+|n_1^2|}\,(|00\rangle + n_1|11\rangle)$. When it is applied to the second qubit of the W state, $|W\rangle = (1/\sqrt{3})(|001\rangle + |010\rangle + |100\rangle)$, we get $\hat{D}_2(|W\rangle) = 1/\sqrt{2+|n_2^2|}\,(|001\rangle + n_2|010\rangle + |100\rangle)$. In other words, the application of $\hat{D}_i$ on a state changes the ith qubit according to the following map: $|0\rangle_i \to |0\rangle_i$ and $|1\rangle_i \to n_i|1\rangle_i$. Note that the final state is obtained normalizing the state obtained after we apply the map.

It is worth mentioning that we called the map described in the previous paragraph a 'disentanglement' operator because the state obtained after its application on a given maximally entangled state does not have the same amount of entanglement as before. We have a decrease on the entanglement content of the original state. It is in this sense that one should understand this terminology.

Applying this operator on each qubit in the telecloning channel results in:

$$|\psi;\{n\}\rangle_{PAC} = A\Big(|0000\rangle + \frac{n_P n_{C_1}}{2}|1010\rangle + \frac{n_A n_{C_1}}{2}|0110\rangle + \frac{n_P n_{C_2}}{2}|1001\rangle + \frac{n_A n_{C_2}}{2}|0101\rangle + n_P n_A n_{C_1} n_{C_2}|1111\rangle\Big)_{PAC}, \quad (6)$$

where

$$A = \Big(1 + \frac{|n_P n_{C_1}|^2}{4} + \frac{|n_A n_{C_1}|^2}{4} + \frac{|n_P n_{C_2}|^2}{4} + \frac{|n_A n_{C_2}|^2}{4} + |n_P n_A n_{C_1} n_{C_2}|^2\Big)^{-1/2}. \quad (7)$$

Note that $\{n\} = \{n_P, n_A, n_{C_1}, n_{C_2}\}$ represents all the 'disentanglement' parameters.

Before we proceed we want to show how the entanglement of the state (6) depends on the values of n_j, where $j = P, A, C_1$, and C_2. This analysis is important since it allows one to connect the efficiency of the protocol to the entanglement of the channel. Furthermore, it also justifies why we have called $\hat{D}_i$ a 'disentanglement' operator.

In order to quantify the entanglement of the channel we employ the global entanglement $E_G^{(1)}$ proposed by Meyer and Wallach [25] and fully discussed and generalized in [26],

$$E_G^{(1)}(|\psi;\{n\}\rangle_{PAC}) = 2\left(1 - \frac{1}{4}\sum_{j=1}^{4}\mathrm{Tr}(\rho_j^2)\right), \quad (8)$$

where ρ_j is the reduced density matrix describing qubit j, obtained tracing out all qubits of the channel but j. One can show [26] that $E_G^{(1)}$ is the mean linear entropy of the qubits belonging to the state (6) and that it is related to the purity of the qubits. For our purposes, $E_G^{(1)}$ is a fairly good multipartite entanglement quantifier [25,26].

The general expression for $E_G^{(1)}$ is too cumbersome and not insightful. Therefore, we show here the most representative cases for n_j real. Whenever all but one of the 'disentangling' parameters are equal to one, or, in other words, whenever we apply $\hat{D}_i$ to only one of the channel's qubits we have,

$$E_G^{(1)}(|\psi;n_j\rangle_{PAC}) = \frac{1+6n_j^2+n_j^4}{2(1+n_j^2)^2}, \quad (9)$$

where $j = P, A, C_1$, or C_2. As depicted in Figure 3 we see that the global entanglement is an increasing function of n_j. When we deal with two free parameters, i.e. n_i and n_j different from one, we have two possibilities. For $(n_i, n_j) = (n_A, n_P) = (n_1, n_2)$ we get,

$$E_G^{(1)} = \frac{8n_j^2 + n_j^4 + n_i^4(1+8n_j^4) + n_i^2(8+38n_j^2+8n_j^4)}{2(2+n_j^2+n_i^2+2n_i^2n_j^2)^2}. \quad (10)$$

On the other hand, for $(n_i, n_j) = (n_A, n_1) = (n_A, n_2) = (n_P, n_1) = (n_P, n_2)$ we get

$$E_G^{(1)} = \frac{2(4+5n_i^2+n_j^2(5+(44+5n_j^2)n_i^2+(5+4n_j^2)n_i^4))}{(5+n_j^2+n_i^2+5n_i^2n_j^2)^2}. \quad (11)$$

Both expressions, however, have a similar behavior. Therefore, in Figure 2 we only plot equation (10). Note that, again, the global entanglement is an increasing function of n_i and n_j.

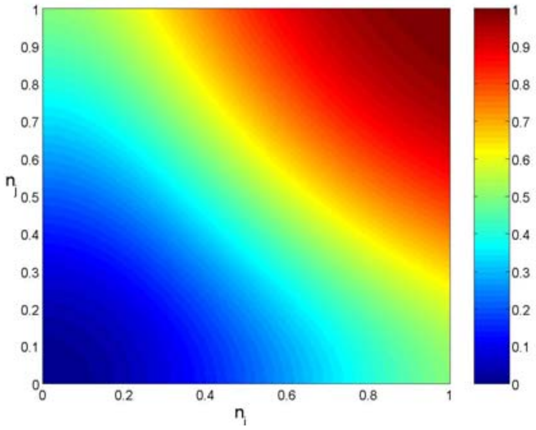

Fig. 2. (Color online) Global entanglement, as given by equation (10), as a function of $(n_i, n_j) = (n_1, n_2) = (n_A, n_P)$.

Let us now return to the telecloning protocol. Using the channel given in equation (6) Alice wants to teleclone an arbitrary state, $|\phi\rangle_X = \alpha|0\rangle_X + \beta|1\rangle_X$, to Bob and Charlie. The full initial state, with the qubit to teleclone, is simply given by

$$|\phi\rangle_{XPAC} = |\phi\rangle_X \otimes |\psi;\{n\}\rangle_{PAC}, \quad (12)$$

and the protocol works as follows.

Alice performs a modified Bell measurement [18,20], i.e. she projects her original (X) and port (P) qubits onto the following modified Bell basis:

$$|\Phi_m^+\rangle = M(|00\rangle + m|11\rangle), \quad (13)$$
$$|\Phi_m^-\rangle = M(m^*|00\rangle - |11\rangle), \quad (14)$$
$$|\Psi_m^+\rangle = M(|01\rangle + m|10\rangle), \quad (15)$$
$$|\Psi_m^-\rangle = M(m^*|01\rangle - |10\rangle), \quad (16)$$

where $M = 1/\sqrt{1+|m|^2}$. We introduce, as will become clear soon, a free parameter (m) in the protocol. It is a proper manipulation of this parameter that allows Alice to overcome the fidelity decrease due to her port qubit disentanglement ($|n_P| < 1$). Each projective measurement implemented by Alice on qubits X and P projects the ancillary and copy qubits to the state $|R_j\rangle_{AC_1C_2}$, with probability P_j. Here $j = \{\Phi_m^+, \Phi_m^-, \Psi_m^+, \Psi_m^-\}$ stands for any possible measurement result obtained by Alice. Alice then sends Bob and Charlie her measurement result (two bits). Then, both parties apply the appropriate unitary transformation on their qubits, $\{\Phi_m^+, \Phi_m^-, \Psi_m^+, \Psi_m^-\} \rightarrow \{I, \sigma_z, \sigma_x, \sigma_z\sigma_x\}$. At the end of the protocol Bob (Charlie) ends up with the state $\rho_{1(2),j} = \mathrm{Tr}_{A,C_{2(1)}}\left(|R_j\rangle_{AC_1C_2}\langle R_j|\right)$, which is obtained tracing out all but qubit $C_1(2)$. Therefore, Bob's (Charlie's) fidelity for this run of the protocol is $F_{1(2),j} = {}_X\langle\phi|\rho_{1(2),j}|\phi\rangle_X$.

3 Channel efficiency

We now turn to estimate the efficiency of the protocol employing the techniques developed in reference [18]. From now on $\{n\}$ and m are all real numbers since it can be shown that we do not lose in generality by such assumptions [18]. In general the probabilities P_j and the fidelities $F_{1(2),j}$ depend on α and β. Moreover, Alice can change the values of α and β of the transferred state at will for each run of the protocol. Therefore, in order to get α- and β-independent results we average over many implementations of the protocol, i.e. over all possible pure state inputs, obtaining the *protocol efficiency* [18]

$$C^{pro}_{1(2)} = \sum_j \langle P_j F_{1(2),j}\rangle.$$

In the averaging process we will need the quantities $\langle|\alpha|^2\rangle$, $\langle|\alpha|^4\rangle$, $\langle|\beta|^2\rangle$, $\langle|\beta|^4\rangle$ and $\langle|\alpha\beta|^2\rangle$. In reference [18] they were shown to be $\langle|\alpha|^2\rangle = \langle|\beta|^2\rangle = 1/2$, $\langle|\alpha|^4\rangle = \langle|\beta|^4\rangle = 1/3$, and $\langle|\alpha\beta|^2\rangle = 1/6$. We can interpret C^{pro} as the average qubit transmission rate for a given protocol choice [18].

The averaged probabilities, Bob's average fidelities, and his channel efficiency are:

$$\langle P_{\Phi^+_m}\rangle = \langle P_{\Psi^-_m}\rangle = \frac{A^2M^2}{2}\Big(1 + \frac{n_P^2 n_{C_1}^2 m^2}{4} + \frac{n_A^2 n_{C_1}^2}{4} + \frac{n_P^2 n_{C_2}^2 m^2}{4} + \frac{n_A^2 n_{C_2}^2}{4} + n_P^2 n_A^2 n_{C_1}^2 n_{C_2}^2 m^2\Big), \quad (17)$$

$$\langle P_{\Phi^-_m}\rangle = \langle P_{\Psi^+_m}\rangle = \frac{A^2M^2}{2}\Big(m^2 + \frac{n_P^2 n_{C_1}^2}{4} + \frac{n_A^2 n_{C_1}^2 m^2}{4} + \frac{n_P^2 n_{C_2}^2}{4} + \frac{n_A^2 n_{C_2}^2 m^2}{4} + n_P^2 n_A^2 n_{C_1}^2 n_{C_2}^2\Big), \quad (18)$$

$$\langle F_{1,\Phi^+_m,\Psi^-_m} P_{\Phi^+_m,\Psi^-_m}\rangle = \frac{A^2M^2}{3}\Big(1 + \frac{n_A^2 n_{C_1}^2}{8} + \frac{n_A^2 n_{C_2}^2}{4} + \frac{n_P n_{C_1} m}{2} + \frac{n_P n_A^2 n_{C_1} n_{C_2}^2 m}{2} + \frac{n_P^2 n_{C_1}^2 m^2}{4} + \frac{n_P^2 n_{C_2}^2 m^2}{8} + n_P^2 n_A^2 n_{C_1}^2 n_{C_2}^2 m^2\Big), \quad (19)$$

$$\langle F_{1,\Phi^-_m,\Psi^+_m} P_{\Phi^-_m,\Psi^+_m}\rangle = \frac{A^2M^2}{3}\Big(m^2 + \frac{n_A^2 n_{C_1}^2 m^2}{8} + \frac{n_A^2 n_{C_2}^2 m^2}{4} + \frac{n_P n_{C_1} m}{2} + \frac{n_P n_A^2 n_{C_1} n_{C_2}^2 m}{2} + \frac{n_P^2 n_{C_1}^2}{4} + \frac{n_P^2 n_{C_2}^2}{8} + n_P^2 n_A^2 n_{C_1}^2 n_{C_2}^2\Big), \quad (20)$$

$$C^{pro}_1 = \frac{2}{3}\left(1 + \frac{1}{2}\frac{\mathcal{F}(\{n\})}{\mathcal{G}(\{n\})}\right), \quad (21)$$

with

$$\mathcal{F}(\{n\}) = (1+n_P^2)(1+n_{C_1}^2)(1+n_A^2 n_{C_2}^2)c(n_P)c(n_{C_1})c(m) - (n_A^2 n_{C_1}^2 + n_P^2 n_{C_2}^2), \quad (22)$$

$$\mathcal{G}(\{n\}) = (n_P^2 + n_A^2)(n_{C_1}^2 + n_{C_2}^2) + 4(1 + n_P^2 n_A^2 n_{C_1}^2 n_{C_2}^2). \quad (23)$$

Here $c(n) = 2n/(1+n^2)$ is the concurrence [27] of the state $1/\sqrt{1+n^2}\,(|00\rangle + n|11\rangle)$. On the other hand, Charlie's fidelities and his channel efficiency are simply obtained by changing $n_{C_1} \leftrightarrow n_{C_2}$. For the standard telecloning protocol $\{n\} = m = 1$ and one obtains the well-known result of $\langle P_j\rangle = 1/4, \langle F_{1(2),j}P_j\rangle = 5/24$, and $C^{pro}_{1(2)} = 5/6$, which is the optimal average fidelity [8].

We now begin to study each qubit's disentanglement effect on the channel efficiency C^{pro}. We investigate how the port, ancillary and copies' disentanglement influence the overall channel efficiency and how we can remedy the disentanglement effect as modelled by equation (5).

3.1 Port qubit treatment

The first qubit we treat is the port. Applying the map giving in equation (5) only to the port qubit (i.e. $n_A = n_{C_{1,2}} = 1.0$) we get:

$$C^{pro}_{1(2)} = \sum_j \langle F_{1(2),j}P_j\rangle = \frac{11}{18}\left(1 + \frac{4c(m)c(n_P)}{11}\right). \quad (24)$$

Note that for $n_P = m = 1$ we obtain $C^{pro}_{1(2)} = 5/6$, the original telecloning efficiency [8]. Moreover, noting that for this case the channel can be written as

$$|\psi;\{n\}\rangle_{PAC} = \frac{1}{\sqrt{1+n_P^2}}\left(|0\rangle_P|\phi_0\rangle_{AC} + n_P|1\rangle_P|\phi_1\rangle_{AC}\right), \quad (25)$$

it is evident to see that the same treatment as in the Generalized Teleportation Protocol (GTP) [18] and the Generalized Quantum State Sharing (GQSTS) [20] applies here. By simply changing the measurement basis (adjusting a proper m) and choosing the proper acceptable measurements one can either retain unit probability of success with low fidelity ($m = 1$, accepting all results), or transform the protocol to a probabilistic one with optimal fidelity (5/6). For example, by choosing $m = n_P$ we recover probabilistically [18,20] the noiseless telecloning protocol [8]. For this choice of m, only $|\Phi^-_m\rangle$ and $|\Psi^+_m\rangle$ are acceptable results both of which furnishing the optimal fidelity for a given run of the protocol (no need for averaging) [18,20].

Finally, we can see that the greater the channel efficiency (Eq. (24)) the greater the channel global entanglement. See Figure 3.

3.2 Ancillary qubit treatment

Applying the map given in equation (5) only to the ancillary qubit (i.e. $n_P = n_{C_{1,2}} = 1.0$) we get

$$C^{pro}_{1(2)} = \sum_j \langle F_{1(2),j}P_j\rangle = \frac{11}{18}\left(1 + \frac{4c(m)}{11}\right). \quad (26)$$

It is interesting to note that the ancillary disentanglement ($n_A < 1$) has no effect on the overall channel efficiency. In other words, equation (26) does not depend on n_A.

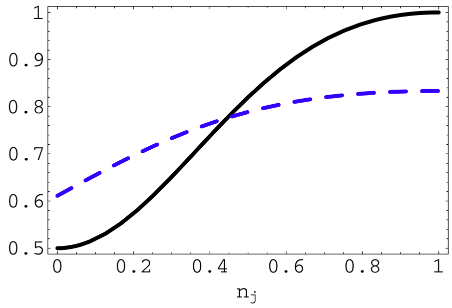

Fig. 3. Global entanglement (black-solid), as given by equation (9), and the channel efficiency (blue-dashed), equation (24), as a function of $n_j = n_P$ for $c(m) = 1$.

It is worth noting that equation (26) is equal to equation (24) when $c(n_P) = 1$, i.e., when one still has a maximally entangled channel ($E_G^{(1)} = 1$). Again we find that the overall channel efficiency is optimal for $m = 1$, namely $C_{1(2)}^{pro} = 5/6$.

3.3 Copy qubit treatment

The last case to consider is the one in which we apply the map to the copies. In this case, we assume that the port and the ancillary qubits are not affected (i.e. $n_P = n_A = 1.0$). The channel efficiency can be rewritten as

$$C_1^{pro} = \tfrac{1}{2}\left(1 + \tfrac{2}{3}\left(\kappa^{(1)} + \kappa^{(2)} c(n_{C_1}) c(m)\right)\right), \qquad (27)$$

$$\kappa^{(1)} = \frac{1}{1+\lambda}, \quad \kappa^{(2)} = \frac{1}{1+1/\lambda}, \qquad (28)$$

$$\lambda = \frac{(1+n_{C_1}^2)(1+n_{C_2}^2)}{1+n_{C_1}^2 n_{C_2}^2}. \qquad (29)$$

For the second copy, C_2^{pro} is given by changing $n_{C_1} \leftrightarrow n_{C_2}$. As we discuss below, equation (27) allows us to derive a couple of interesting properties for this particular protocol. Firstly, let us analyze some trivial limiting cases. For $m = 1$, note that when $n_{C_1} = n_{C_2} = 1$ we obtain, as it should be, $C_{1(2)}^{pro} = 5/6$, the noiseless optimal limit. Moreover, when $n_{C_1} = n_{C_2} = 0$ we get $C_{1(2)}^{pro} = 2/3$. This value can be understood noting that for this case the channel is $|\psi; \{n\}\rangle_{PAC} = |0000\rangle_{PAC}$, i.e. we have no entanglement whatsoever. Thus the telecloning protocol can be seen as the usual teleportation protocol whose efficiency is at most 2/3 when we have pure but not entangled channels [18]. Furthermore, only for the case when $n_{C_1} = 1$, we see that the channel efficiency of the first copy does not depend on n_{C_2}, as can be seen looking at equation (29). A similar argument applies for the second copy channel efficiency. This is remarkable and it means that the application of the map on the second (first) copy changes the protocol efficiency of the first (second) copy in a way that depends on the application of the map on the first (second) copy. Finally, when $n_{C_2} = 1$ one recovers $C_1^{pro} = \frac{11}{18}(1 + \frac{4}{11} c(m) c(n_{C_1}))$, similar to equation (24), with $n_P \leftrightarrow n_{C_1}$. This shows that

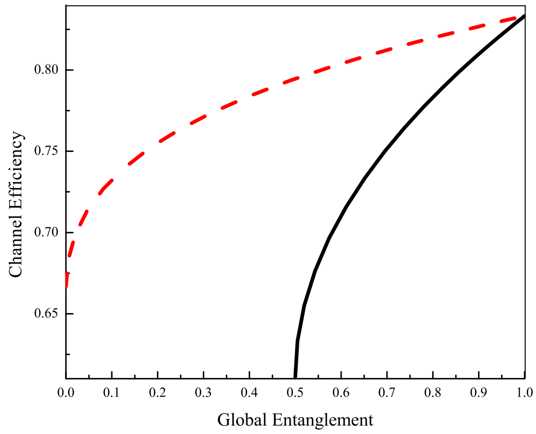

Fig. 4. Channel efficiencies as a function of global entanglement when the 'disentanglement' map is applied to the port (black-solid) and to the copies (red-dashed), as given by equations (24) and (27) ($n_{C_1} = n_{C_2}$ or $n_A = n_P$), respectively. In all cases $c(m) = 1$.

the action of the map on the port qubit ($n_P < 1$) changes C_1^{pro} in exactly the same way as when the map acts on just the first copy ($n_{C_1} < 1$). However, in contrast to the case where the map acts only on the port qubit, we were not able to devise a procedure by which we can increase the fidelity of the copies, even in a probabilistic protocol. In other words, equating $m = n_{C_1}$ does not improve the fidelity of the copies, contrary to a similar successful strategy ($m = n_P$) employed for the port qubit case.

We end this section showing that the channel efficiency is a monotonic increasing function of the global entanglement, as depicted in Figure 4.

4 GTC to GTP conversion

We end this article showing how one can convert the GTC to the GTP protocol. In other words, we want to show how it is possible, using first local and then global unitary operations, to convert the GTC channel $|\psi; \{n\}\rangle_{PAC}$ (Eq. (6)) to the GTP channel $|\Psi_{n_{C_1}}^{GTP}\rangle = \left(1/\sqrt{1+n_{C_1}^2}\right)$ $(|00\rangle + n_{C_1}|11\rangle)$. We want, therefore, to create a GTP channel between Alice and copy 1 (Bob) in detriment of copy 2 (Charlie), who will have a considerable decrease of his channel efficiency. This can be achieved by 'disentangling' copy 2 from Alice's qubit and copy 1. The final goal is to concentrate all the entanglement of the channel between Alice and Bob.

4.1 Local unitary operations

Firstly, let us restrict ourselves to local unitary operations (Alice's site). If we remember that the ancillary qubit (A) is assumed to be with Alice, she can only operate on the port (P) and ancillary qubits (see Fig. 1). An optimal strategy for Alice, when we set $n_P = n_A = 1$, $n_{C_2} = 0$, and $m = 1$ for the measurement basis, is the application of the following unitary operation on A and P:

$$\boldsymbol{R}_{jk}(q) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & A_q & -qA_q & 0 \\ 0 & q^* A_q & A_q & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad (30)$$

$$A_q = \frac{1}{\sqrt{1+|q|^2}}, \quad (31)$$

where $j, k = P, A$ are the two qubits Alice acts upon. Here $\boldsymbol{R}_{jk}$ is written in the basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ and it is basically a rotation in the $|01\rangle, |10\rangle$ plane. The best result (maximal channel efficiency) is achieved for the case $q = 1$. This choice for q gives the channel (note the order in which the qubits are written),

$$|\Psi^{GTP}_{n_{C_1}/\sqrt{2}}\rangle_{PC_1AC_2} = \frac{1}{\sqrt{1+n_{C_1}^2/2}} \times \left(|00\rangle + \frac{n_{C_1}}{\sqrt{2}}|11\rangle\right) \otimes |00\rangle. \quad (32)$$

This is a GTP-like channel between P and C_1 but with $n_{C_1}/\sqrt{2}$ instead of n_{C_1}, which is the cost one pays for the inaccessibility to the copy qubits. However, the channel efficiency is still large since for $n_{C_1} = 1$ we have $C_1^{pro} = (6 + 2\sqrt{2})/9 \approx 0.981$. Furthermore, we can also implement with the above channel a probabilistic teleportation protocol. This means we can have, sometimes, a unity fidelity teleported state, i.e. a successful run of the protocol [18,19].

Borrowing from the case of $n_{C_2} = 0$ and any n_{C_1}, to the case of $n_{C_1} = 1$ and $n_{C_2} < 1$, we can make the same transformations as before and arrive at the following channel efficiencies:

$$C_1^{pro} = \frac{6 + 2\sqrt{2} + 5n_{C_2}^2}{9\,(1 + n_{C_2}^2)}, \quad (33)$$

$$C_2^{pro} = \frac{5 + 2\sqrt{2}n_{C_2} + 6n_{C_2}^2}{9\,(1 + n_{C_2}^2)}. \quad (34)$$

Looking at equations (33) and (34) we can draw several interesting conclusions: (i) $C_1^{pro} > C_2^{pro}$ for all n_{C_2}, which is a consequence of the fact that Alice's qubit is more entangled with copy 1 qubit, located at Bob's, in comparison with copy 2 at Charlie's; (ii) for $n_{C_2} = 1$ we get $C_1^{pro} < 5/6$ and $C_2^{pro} < 5/6$, showing that the unitary transformation reduces the channel efficiency of the GTC protocol when compared with the efficiency of a maximally entangled GTC channel; (iii) equations (33) and (34), however, show that for $n_{C_2} \le \sqrt{\frac{4\sqrt{2}-3}{5}}$ one can achieve $C_1^{pro} \ge 5/6$, thus highlighting the transition point from the GTC to the GTP scenario.

4.2 Global unitary operations

If we now allow Alice to implement global unitary operations, i.e., she has access, in addition to the port and ancillary qubits, to at least one of the copies, she is able to recover the GTP channel from the GTC channel via two transformations. We also assume, from now on, that Alice has access only to copy 1, being, thus, impossible for her to work with copy 2.

As we did before, we first assume that $n_P = n_A = 1$, $n_{C_2} = 0$, and $m = 1$ for the measurement basis. With this choice, the GTC channel reads,

$$|\psi; \{n\}\rangle_{PAC_1C_2} = \frac{1}{\sqrt{4 + 2n_{C_1}^2}}\Big(2|000\rangle + n_{C_1}|101\rangle + n_{C_1}|011\rangle\Big) \otimes |0\rangle. \quad (35)$$

First Alice implements the following unitary operation on the ancillary and copy 1 qubits, setting $q = n_{C_1}/2$,

$$\boldsymbol{T}_{jk}(q) = \begin{pmatrix} A_q & 0 & 0 & qA_q \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -q^* A_q & 0 & 0 & A_q \end{pmatrix}, \quad (36)$$

$$A_q = \frac{1}{\sqrt{1+|q|^2}}, \quad (37)$$

where j, k are the two qubits Alice acts upon and now $\boldsymbol{T}$ is basically a rotation in the $|00\rangle, |11\rangle$ plane. The resulting state, $|\Psi^{(1)}\rangle = \boldsymbol{T}_{A,C_1}\left(\frac{n_{C_1}}{2}\right)|\psi; \{n\}\rangle_{PAC_1C_2}$, is,

$$|\Psi^{(1)}\rangle_{PAC_1C_2} = \frac{1}{\sqrt{4 + 2n_{C_1}^2}}\left(\sqrt{4 + n_{C_1}^2}|0000\rangle + n_{C_1}|1010\rangle\right). \quad (38)$$

The second transformation Alice implements are on the port and copy 1 qubits with $q = n_{C_1}(1 - \sqrt{4 + n_{C_1}^2})/(n_{C_1}^2 + \sqrt{4 + n_{C_1}^2})$. The final state,

$$|\Psi^{GTP}\rangle_{PC_1AC_2} = \boldsymbol{T}_{P,C_1}\left(\frac{n_{C_1}(1 - \sqrt{4 + n_{C_1}^2})}{n_{C_1}^2 + \sqrt{4 + n_{C_1}^2}}\right)|\Psi^{(1)}\rangle,$$

is given as,

$$|\Psi^{GTP}\rangle_{PC_1AC_2} = \frac{1}{\sqrt{1 + n_{C_1}^2}}\,(|00\rangle + n_{C_1}|11\rangle) \otimes |00\rangle. \quad (39)$$

This is exactly the GTP channel [19,18] we were looking for. Therefore, if Alice has also access to copy 1, it is possible to go from GTC to GTP.

Again, borrowing from the case in which $n_{C_2} = 0$ and n_{C_1} is the free parameter, to the case of $n_{C_1} = 1$ and $n_{C_2} < 1$, we can make the same transformations as before and arrive at the following channel efficiencies:

$$C_1^{pro} = \frac{135 + 77n_{C_2}^2}{135(1 + n_{C_2}^2)}, \quad (40)$$

$$C_2^{pro} = \frac{135 + (8\sqrt{5} + 159)n_{C_2}^2 + 24\sqrt{15}n_{C_2}}{270(1 + n_{C_2}^2)}. \quad (41)$$

Here, again, we have the following interesting results: (i) $C_1^{pro} > C_2^{pro}$ for all n_{C_2}, reflecting the concentration of entanglement between port and copy 1; (ii) for $n_{C_2} = 1$ we get $C_1^{pro} < 5/6$ and $C_2^{pro} < 5/6$, showing that the transformations also reduce the channel efficiency of the GTC protocol when compared with the efficiency for the maximally entangled GTC channel; (iii) finally, manipulating C_1^{pro}, one sees that for $n_{C_2} \leq \sqrt{45/71}$ one can achieve $C_1^{pro} \geq 5/6$, thus showing the transition point from the GTC to the GTP scenario.

5 Experimental proposal

The main experimental challenge in order to implement the GTC is the ability of Alice to apply on her qubits a generalized Bell measurement. In other words, Alice must project the port qubit (P) and the one to be telecloned (X) onto one of the four generalized Bell states given in equations (13–16). Fortunately, this can be achieved for the following qubit encodings [28]: (i) single-photon state and the vacuum state; (ii) a vertically and a horizontally polarized photon state; and (iii) two coherent light states with opposite phases. Using linear optical schemes Kim et al. [28] have shown how one is able to implement a generalized Bell measurement for each one of the above three possible qubit encodings. For the first two encodings, not all generalized Bell states can be distinguished via linear optics, although the last one allows an almost perfect discrimination among the four generalized Bell states.

6 Conclusion

To conclude, we have shown that decreasing the entanglement of the quantum channel needed for a perfect quantum telecloning protocol results in non-trivial protocol efficiencies which depend on the specific mechanism used to decrease its entanglement content ('disentanglement' process). We have analyzed all the three possible 'disentanglement' scenarios. Firstly, acting locally on the port qubit, the reduction of the channel's entanglement can be dealt with in a probabilistic manner, similar to the approach employed for the generalized teleportation and quantum state sharing protocols. Here we can achieve the optimal fidelity for the telecloned qubits by properly rotating Alice's measurement basis. Secondly, the ancillary's disentanglement has no effect on the overall average efficiency, as expected from an ancillary. Thirdly, the copies' disentanglement cannot be counter attacked using the port's disentanglement approach, i.e., there is no rotation on Alice's measurement basis allowing, even probabilistically, the optimal fidelity for both telecloned qubits. Finally, we have also shown how one can convert the generalized telecloning channel, either using local or global unitary operations, to the generalized teleportation channel. All these results highlight that non-maximally pure entangled channels can also be employed to the direct implementation of quantum telecloning, although only probabilistically. And this suggests that a promising route for further analysis is the study of what can be done probabilistically using directly, i.e. without distillation protocols, non-maximally mixed entangled channels.

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